Geometry Syllabus 2012/2013/2014

Strand 1: Geometry and Trigonometry

Students learn about	St[illegible] working at FL [illegible]ould be able to	In addition, students working at OL should be able to	In addition, st[illegible] working at HL should be able [illegible]
2.1 [illegible]**nthetic** [illegible]**metry** *	– perform constructions **18, 19, 20** (see *Geometry Course for Post-primary School Mathematics*)	– perform constructions **16, 17, 21** (see *Geometry Course for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies – investigate theorems **7, 8, 11, 12, 13, 16, 17, 18, 20, 21** and **corollary 6** (see *Geometry Course for Post-primary School Mathematics*) and use them to solve problems	– perform constructions **1-15** and **22** (see *Geometry Course for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction – prove theorems **11, 12, 13,** concerning ratios (see *Geometry Course for Post-primary School Mathematics*), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle
2.2 Co-ordinate geometry	– use slopes to show [illegible] lines are • parallel • perpendicular	[illegible] triangle – recognise the fact that the relationships $y = mx+c$, $y-y_1 = m(x-x_1)$ and $ax + by + c = 0$ are linear – solve problems involving slopes of lines – recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle centre (h, k) and radius r – solve problems involving a line and a	– solve problems involving • the perpendicular distance from a point to a line • the angle between two lines – divide a line segment in a given ratio m:n – recognise that $x^2+y^2+2gx+2fy+c = 0$ represents the relationship between the x and y co-ordinates of points on a circle centre $(-g,-f)$ and radius r where $r = \sqrt{g^2+f^2-c}$ – solve problems involving a line and a circle

Section B Contexts and Applications

Section A Concepts and Skills

"In the examination, candidates will have the option of answering a question on the synthetic geometry set out here,

or

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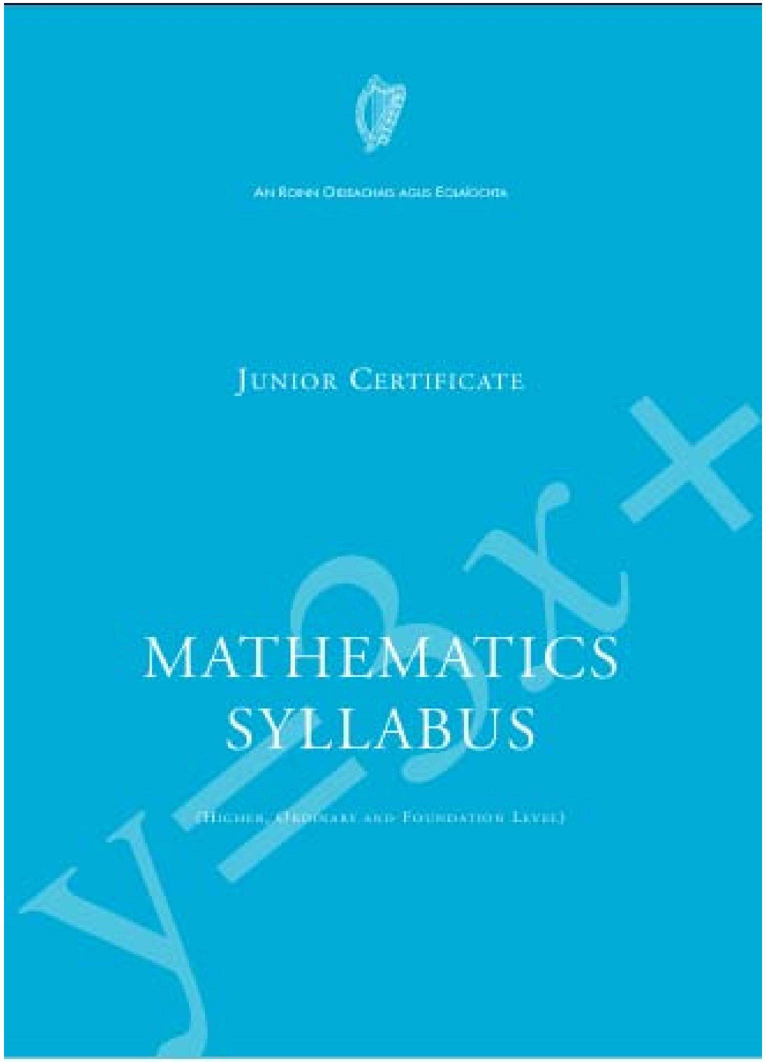
Geometry Syllabus 2012/2013/2014



Section A	Concepts and Skills

or

answering a problem solving question based on the geometrical results from the corresponding syllabus level at Junior Certificate." (pg 22)

Section B	Contexts and Applications

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Section A: Geometry 2012/2013/2014

Higher

- Constructions 1 – 15 and 22
- Terms: is equivalent to, if and only if, proof by contradiction
- Prove theorems 11,12,13,

Ordinary

- Constructions 16, 17, 21
- Terms: theorem, proof, axiom, corollary, converse, implies
- Investigate and use theorems 7,8,11,12,13,16, 17,18,20,21 and corollary 6

Foundation

- Constructions 18, 19,20

Proceed With Caution!

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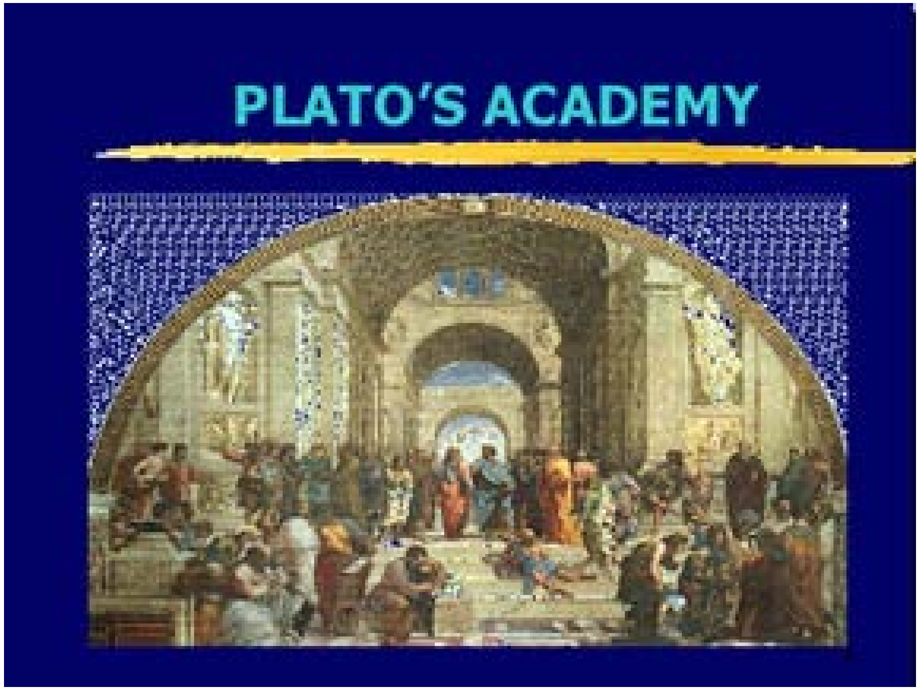
Proofs which were previously acceptable on the old Mathematics Syllabus may not necessarily be acceptable now if they do not fit within the logical framework of the Geometry Course for Post – primary School Mathematics.

The Student’s CD aims to supply approved proofs for all the theorems but does not propose that only these proofs will be acceptable.

Any variant proofs should be referred to the State Examinations Commission (SEC) and/or National Council for Curriculum & Assessment (NCCA) to see if they are acceptable.

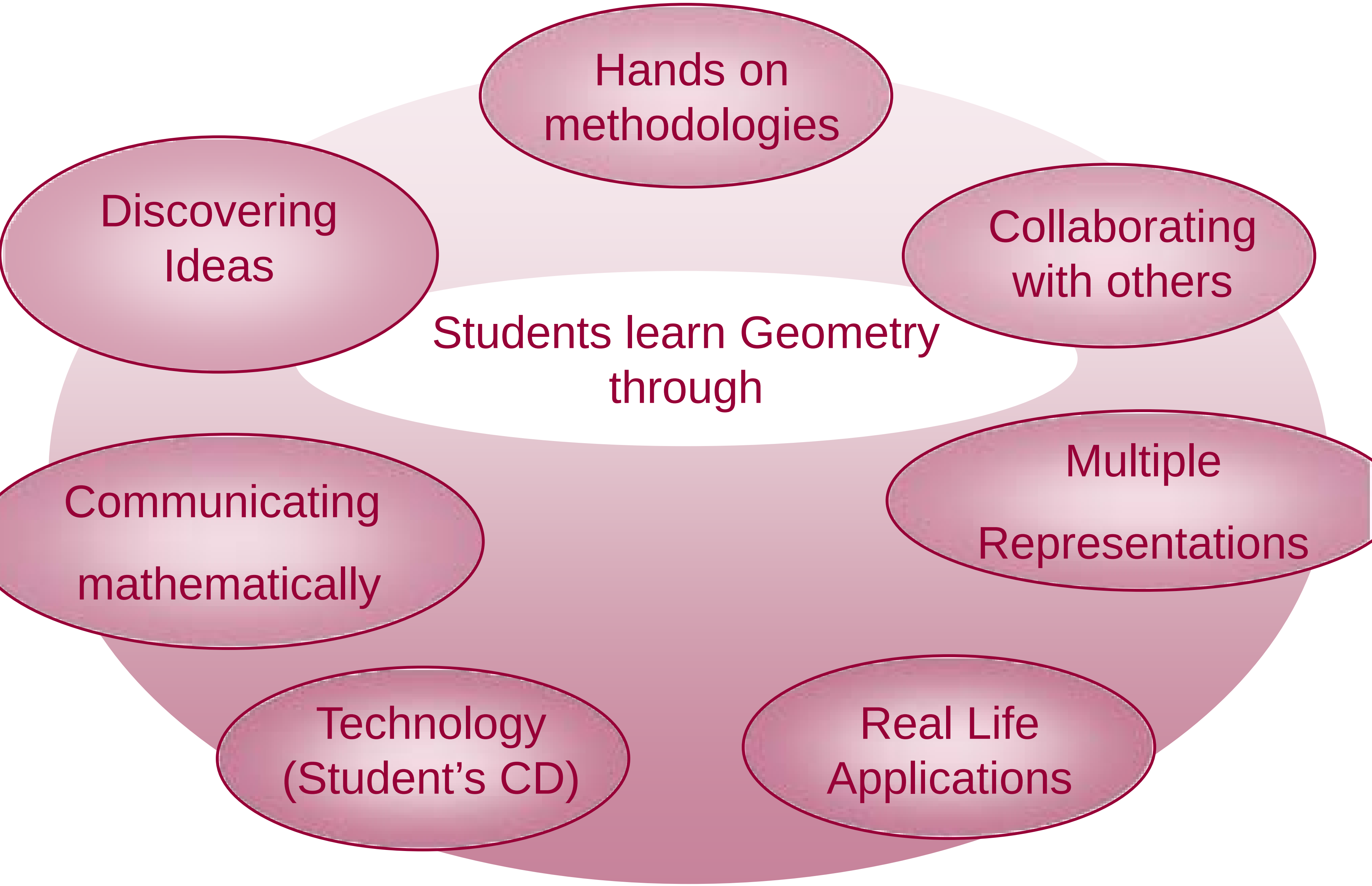
(NCCA, Leaving Certificate Syllabus. (2012) p33).

Plato’s Academy



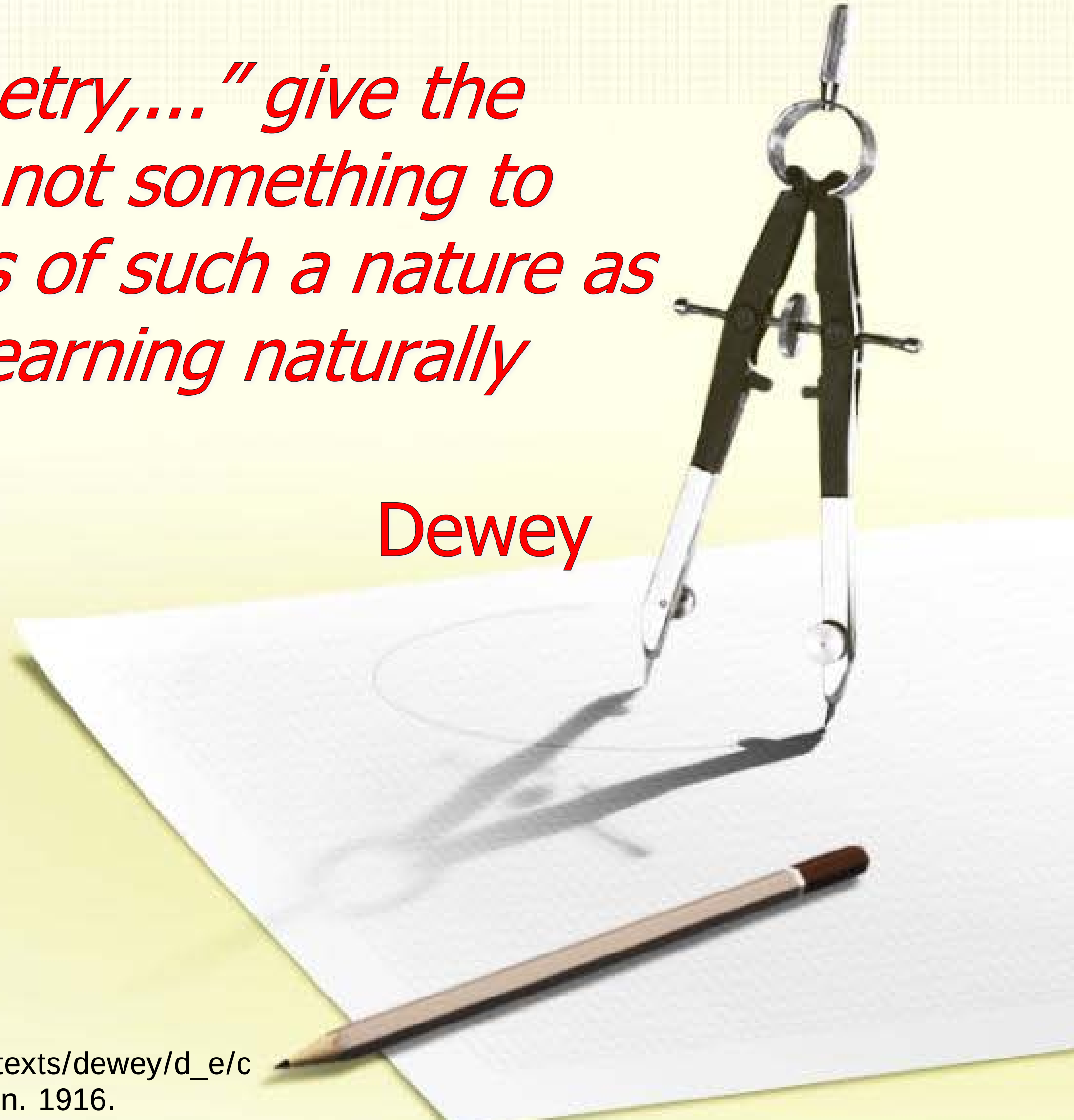
ἀγεωμὲρητος
μηδεὶς
εἰσίτω

“Let no-one who is ignorant of Geometry enter here”


Students learn Geometry through
Hands on methodologies
Collaborating with others
Multiple Representations
Real Life Applications
Technology (Student’s CD)
Communicating mathematically
Discovering Ideas

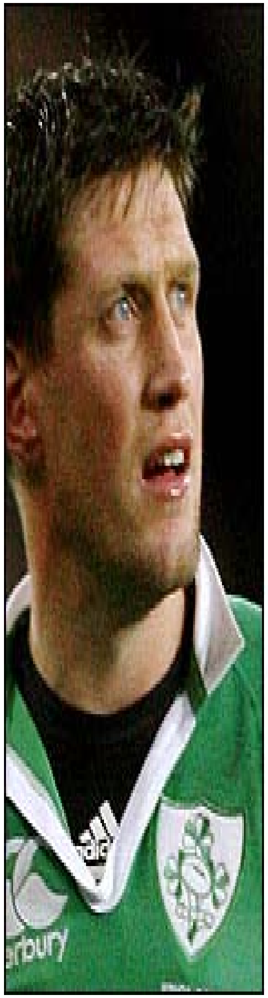
When it comes to Geometry,..." give the pupils something to do, not something to learn; and if the doing is of such a nature as to demand thinking;... learning naturally results."

Dewey



[Source: http://www.ilt.columbia.edu/publications/Projects/digitexts/dewey/d_e/chapter12.html , John Dewey. Democracy and Education. 1916.

Geometry around us



NYC Geometry

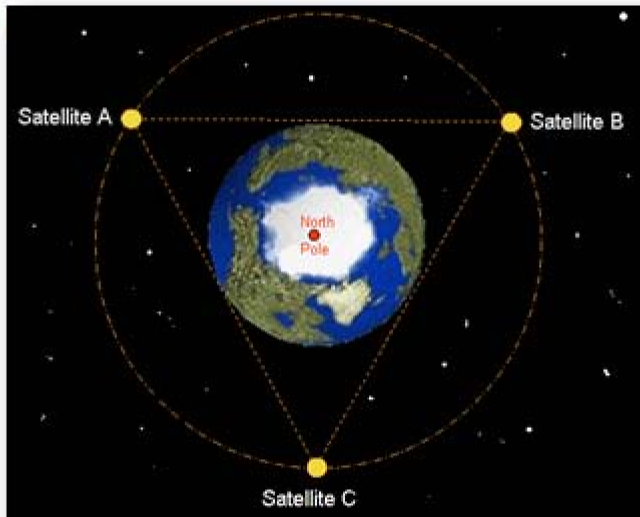
NYC Geometry

Constructions

Theorems & Corollaries

Satellites

1



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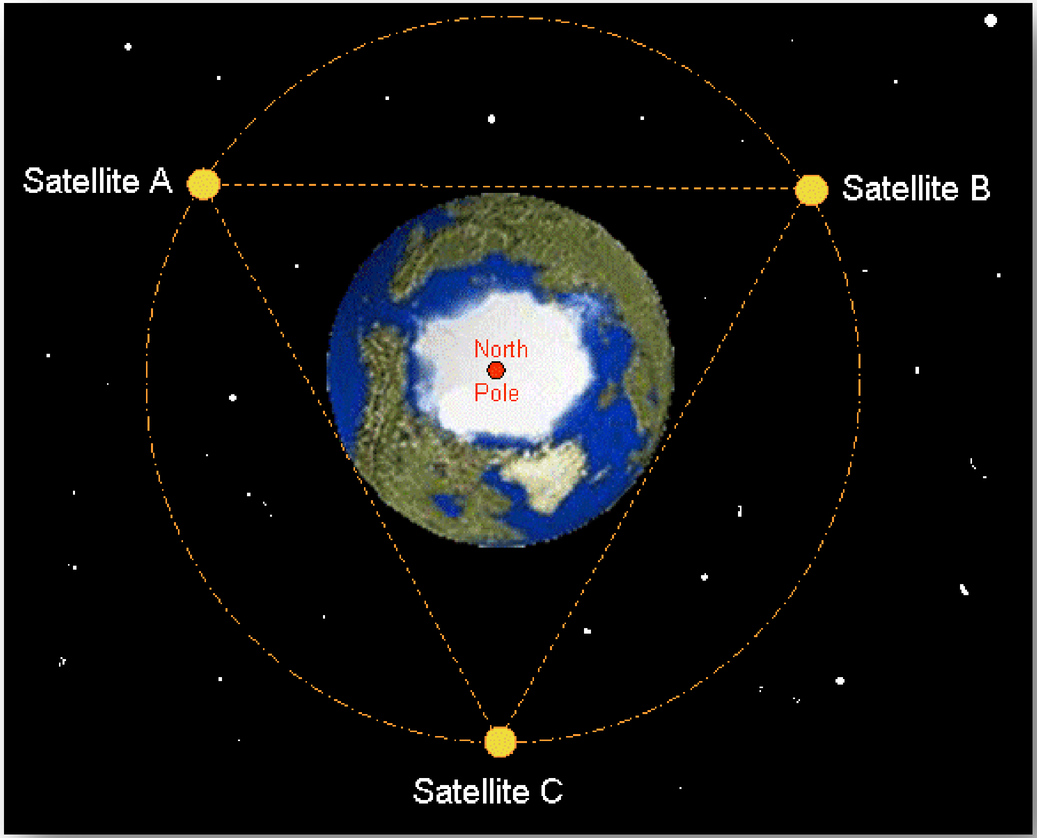
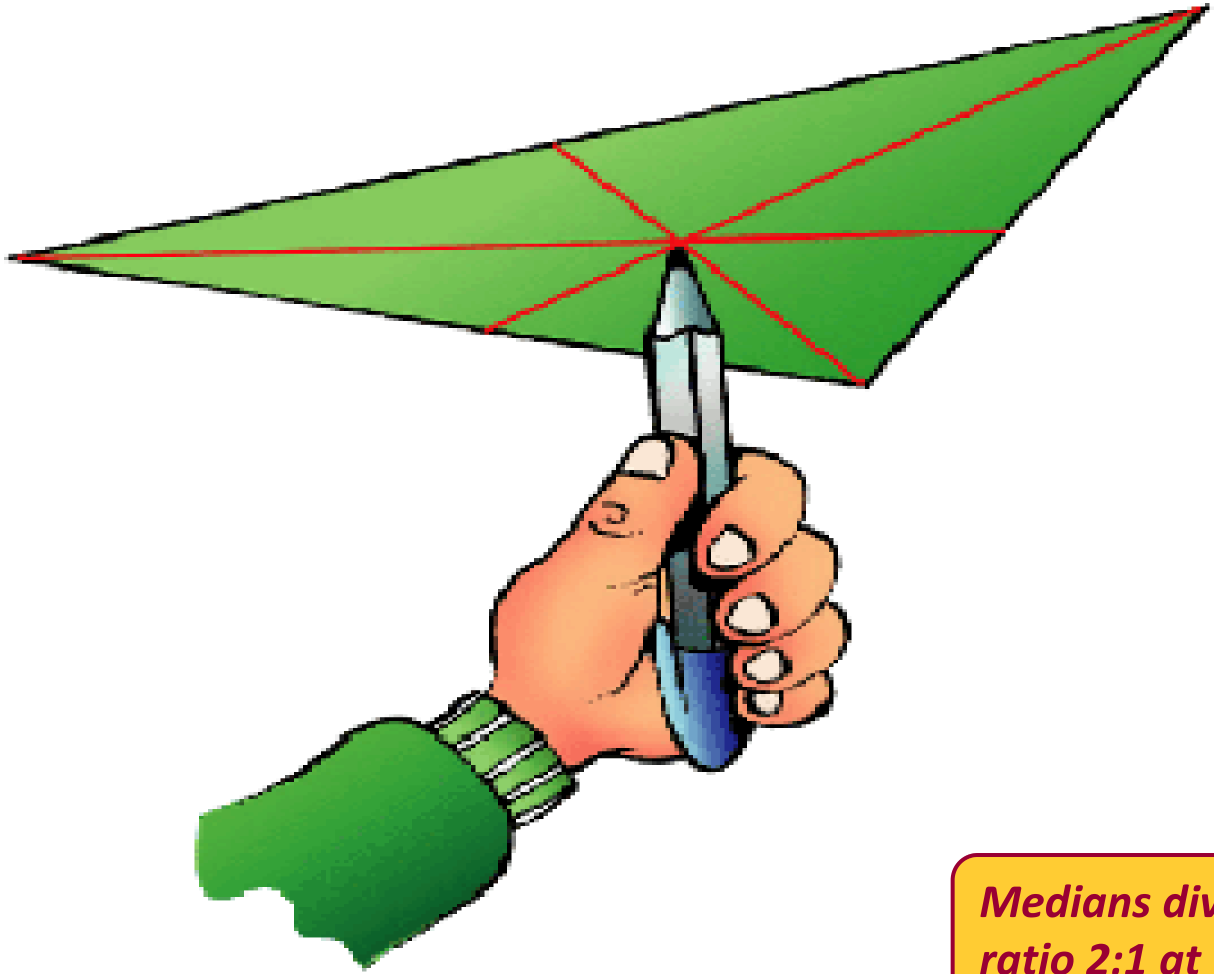
2 & 3

Satellites



Centre of Gravity



Medians divided in the ratio 2:1 at the centroid.

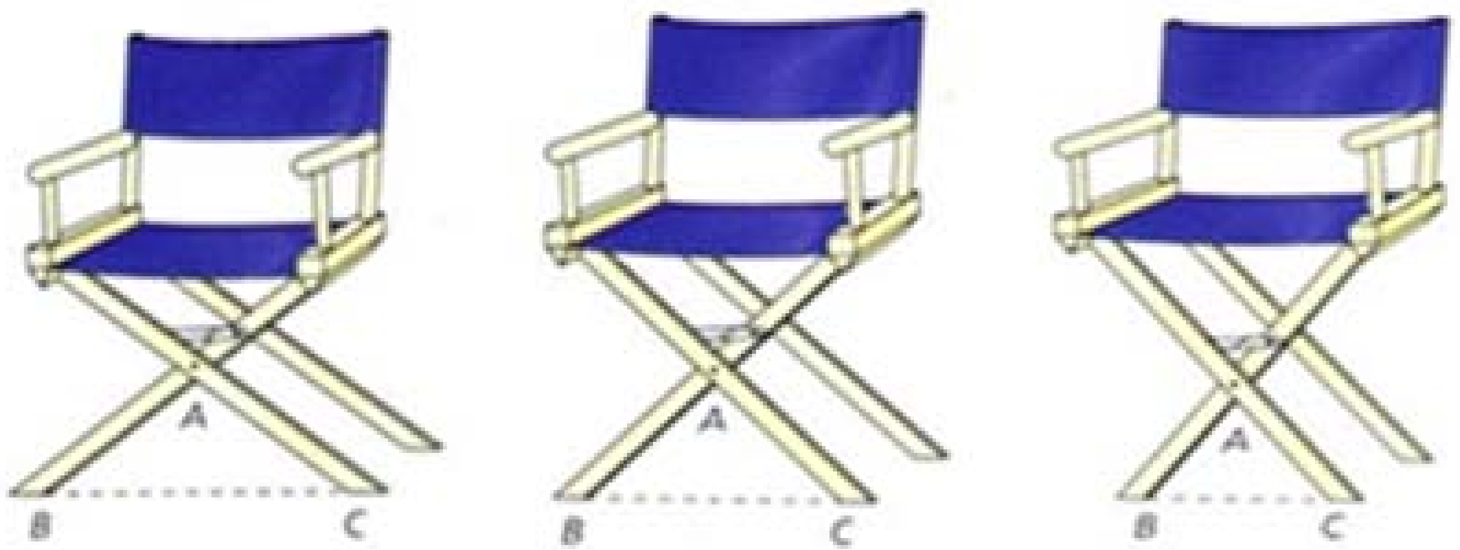
Quickest Route??

Neil needs to get from Bermuda to Miami as quickly as possible. Should he fly direct or could Bermuda-Puerto Rico-Miami be a faster route?

A walk in Melbourne

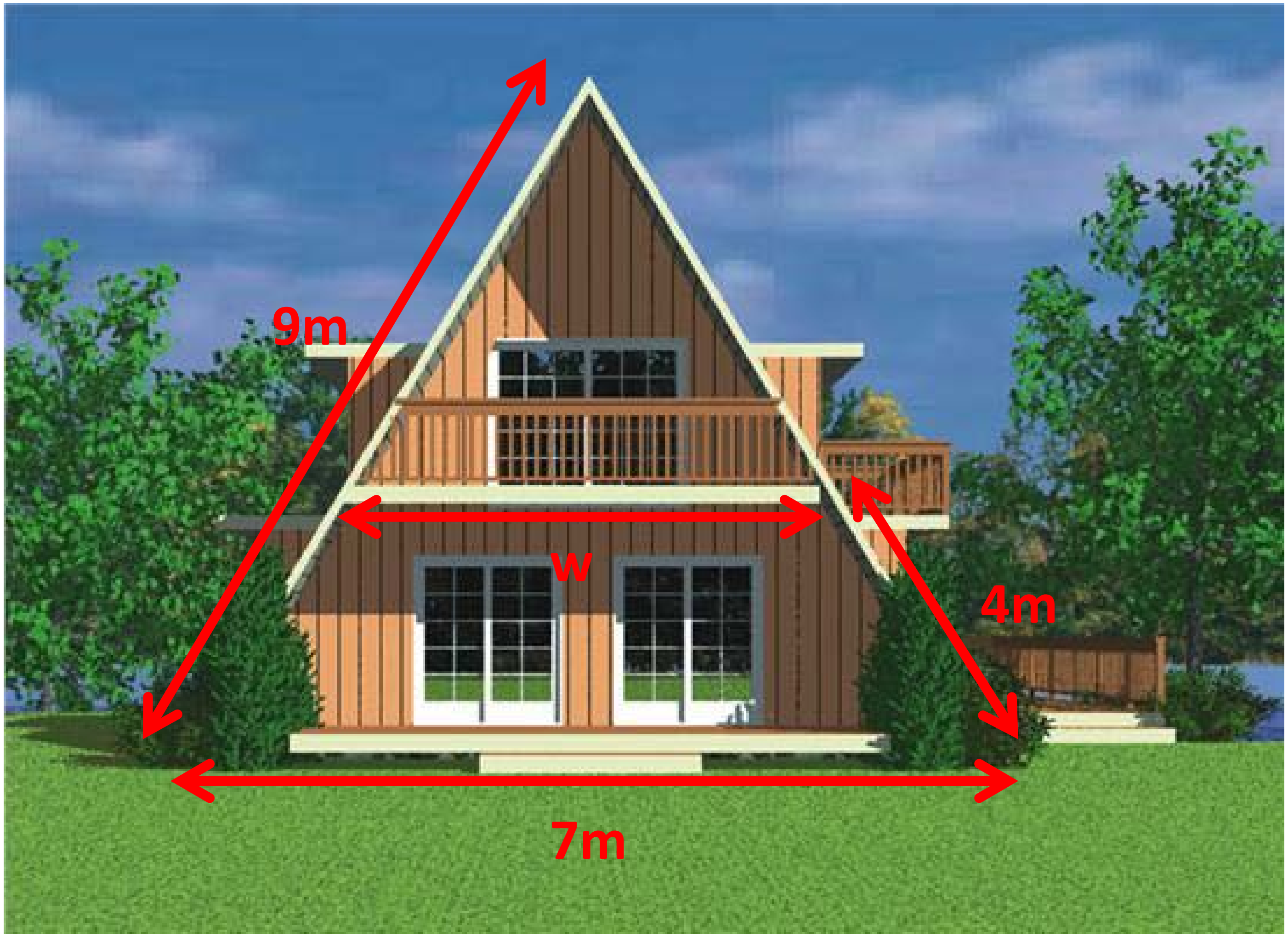
How far is it from Elizabeth St. to Russell St.?

Director's Chair



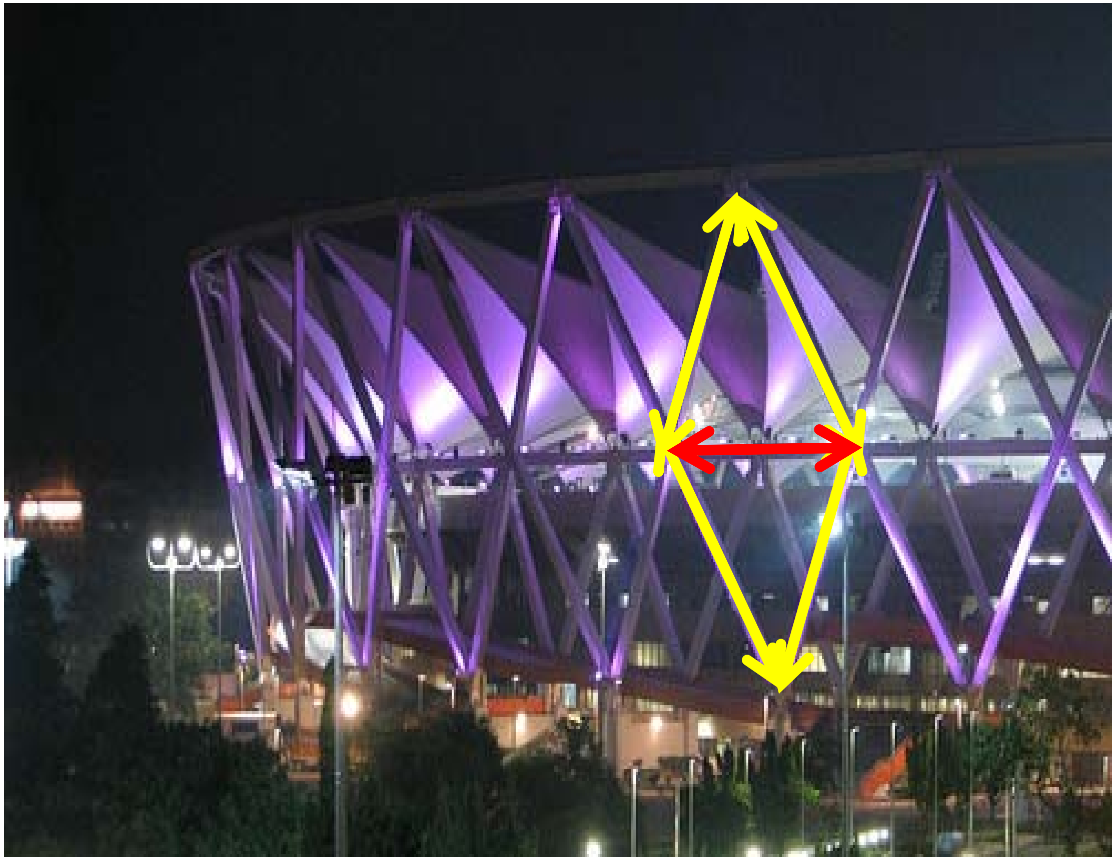
Which chair is most stable? Why?

“A-frame” house

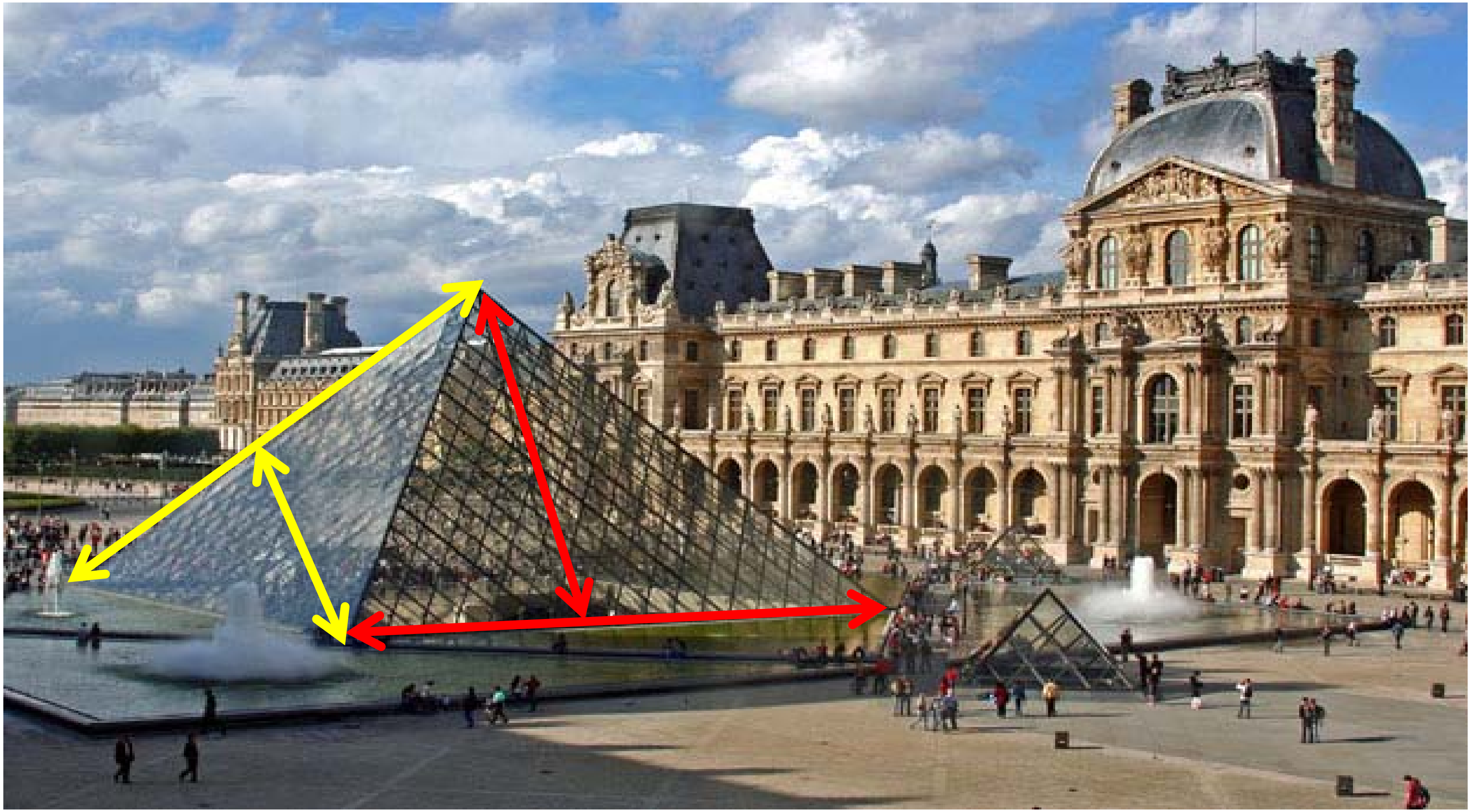


The perfect golf shot!

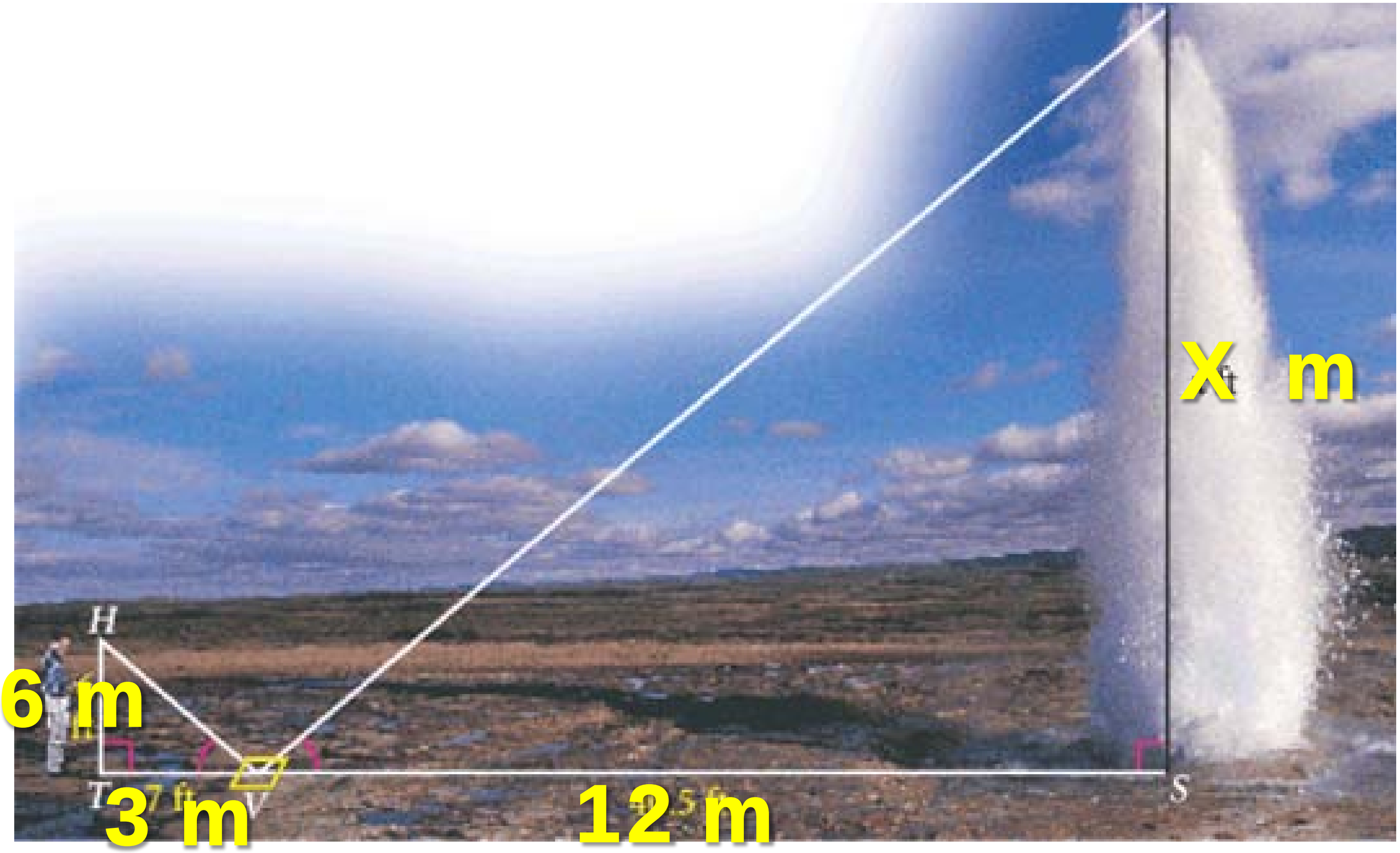
Nehru stadium India



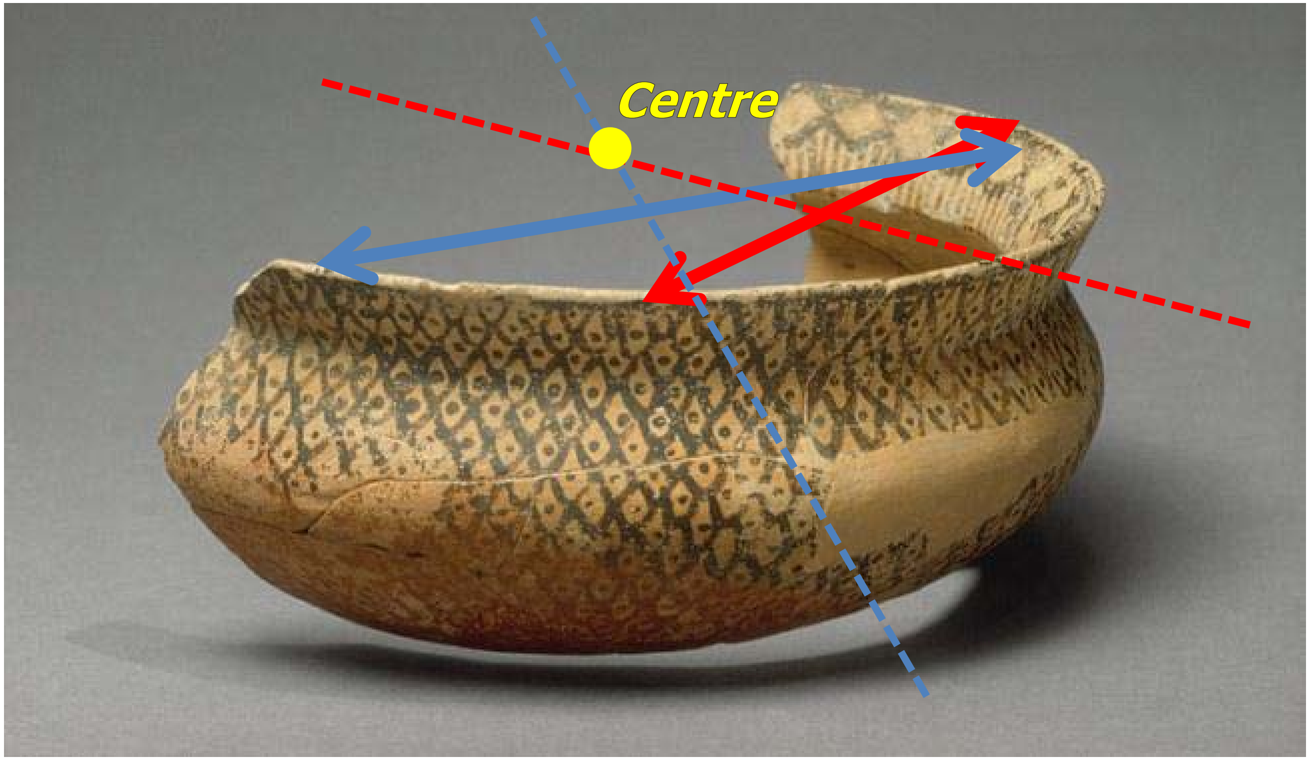
Louvre Entrance



Height of the Geyser



Challenge: Geometry in Action



Can you find the circumference of the circular rim of the bowl using geometrical ideas from your course?

Can you do it in <u>two</u> different ways?

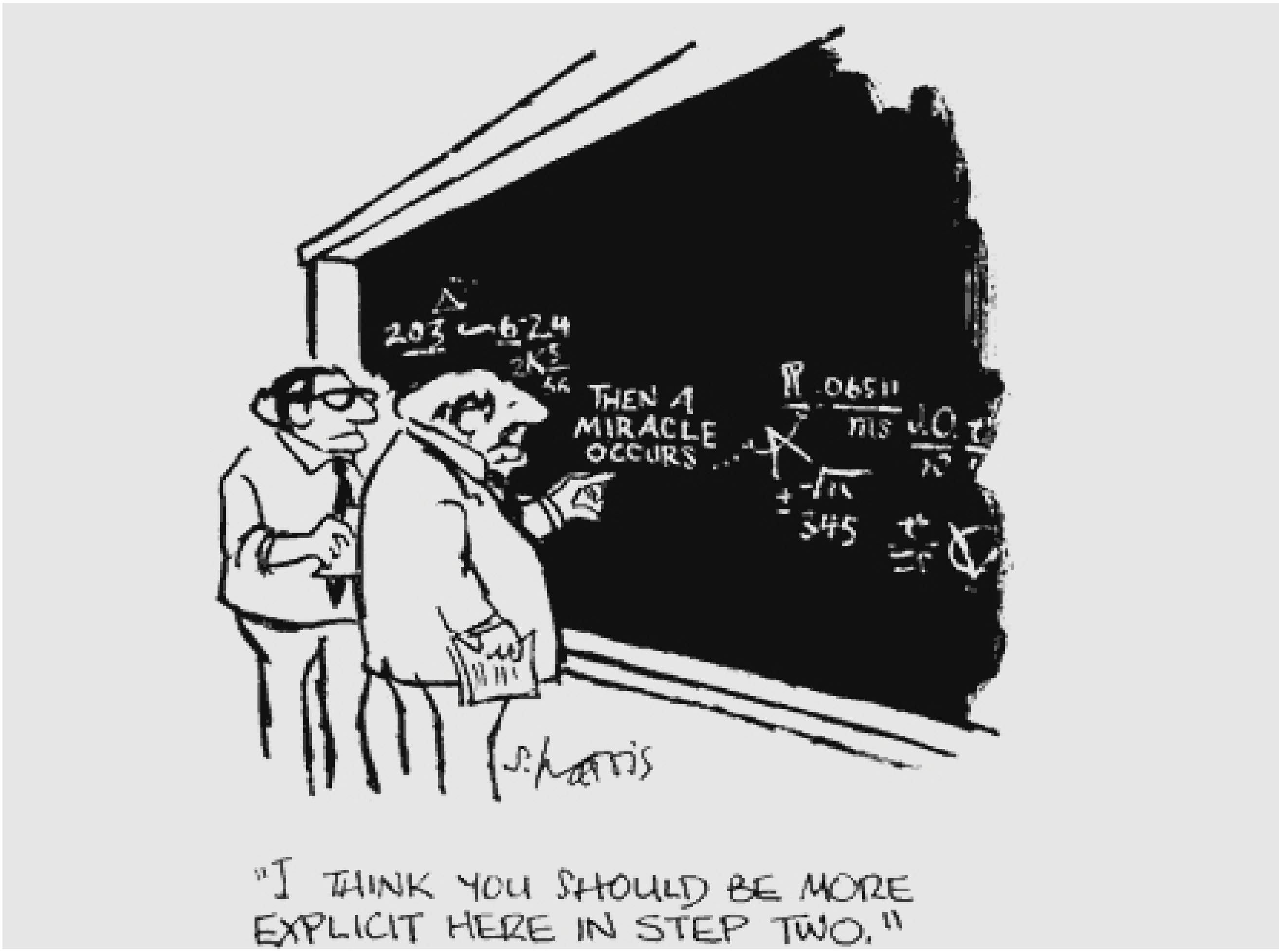
Mathematical Reasoning and Proof

The Master of Deduction

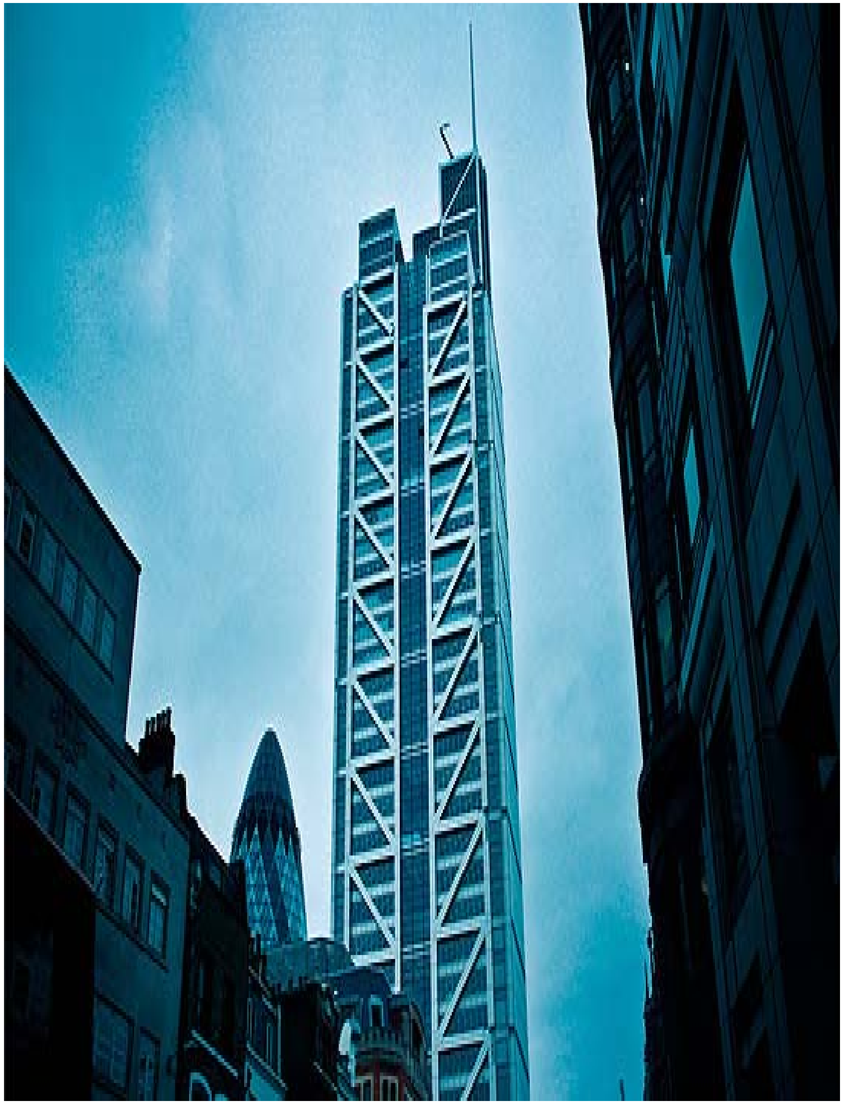
"Sometimes Watson, proof doesn't come easily........"

13:31

The Master of Deduction



Inductive vs Deductive reasoning



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Patterns : Inductive/Deductive reasoning

Can you use the patterns above to work out how many beams are needed for a tower of 12 storeys?

Callan has deduced that the formula for the number of beams (B) needed for a tower with (S) storeys is given by

$$B = 2(S+1) + 3S + S$$

His teacher says he has made 1 mistake in his deduction.
Can you find Callan's error?

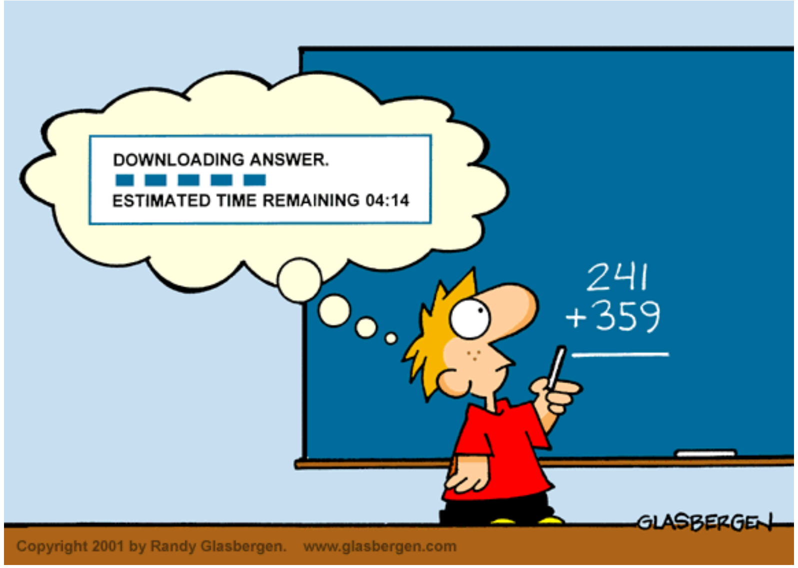
LC HL 2008

Show that if a and b are non-zero real numbers, then the value of

$\frac{a}{b} + \frac{b}{a}$ can never lie between -2 and 2.

Let students discover that empirical argument doesn't mean proof!!

- *Pick any two odd numbers.*
- *Add them.*
- *Do this a few times.*
- *Do you notice anything about your answers?*
- *Do you think this is always true?*

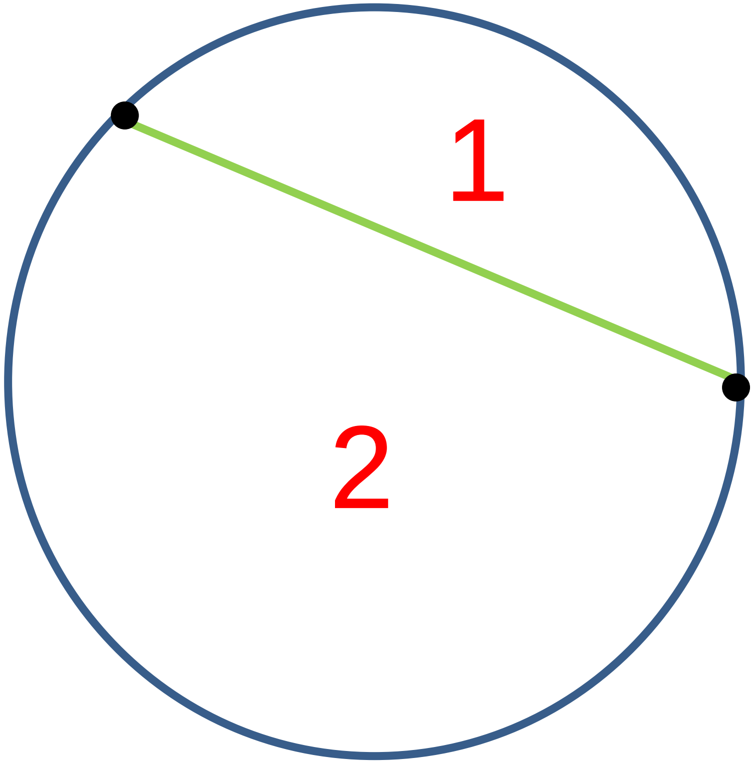


The Regions in a Circle problem

Place different numbers of points around a circle and join each pair. Can you see any relationship between the number of points and the maximum number of regions produced?

Workshop Booklet: Page 4

The Regions in a Circle problem



i.e. Joining 2 points on the circumference of a circle with a chord generates a maximum of 2 regions

NB "Maximum" : No 3 chords concurrent

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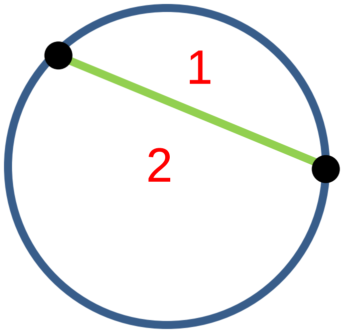

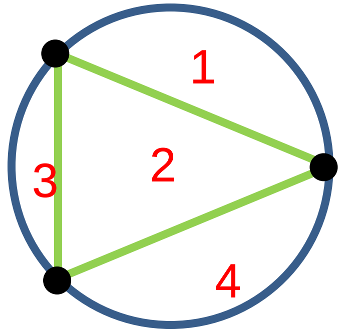

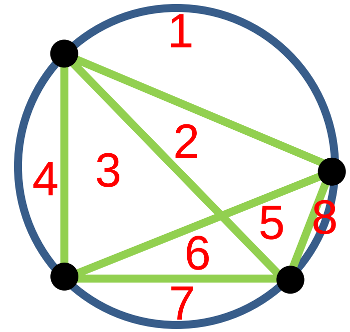

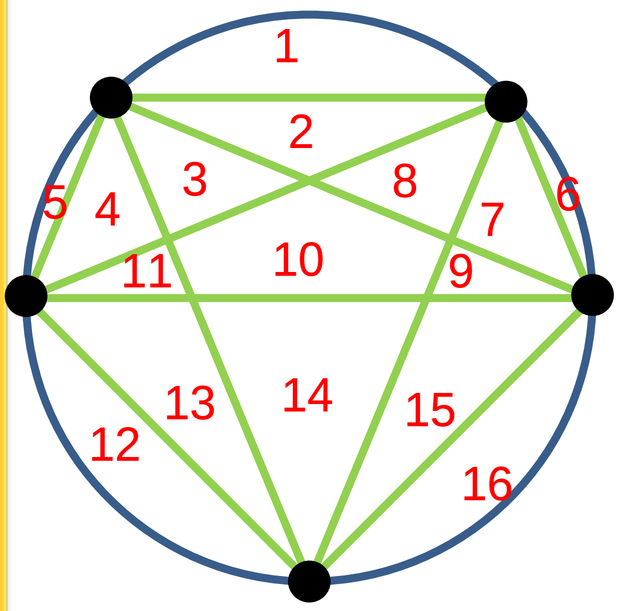

Solutions	
Points	Regions
2	2
3	4
4	8
5	16
6	?

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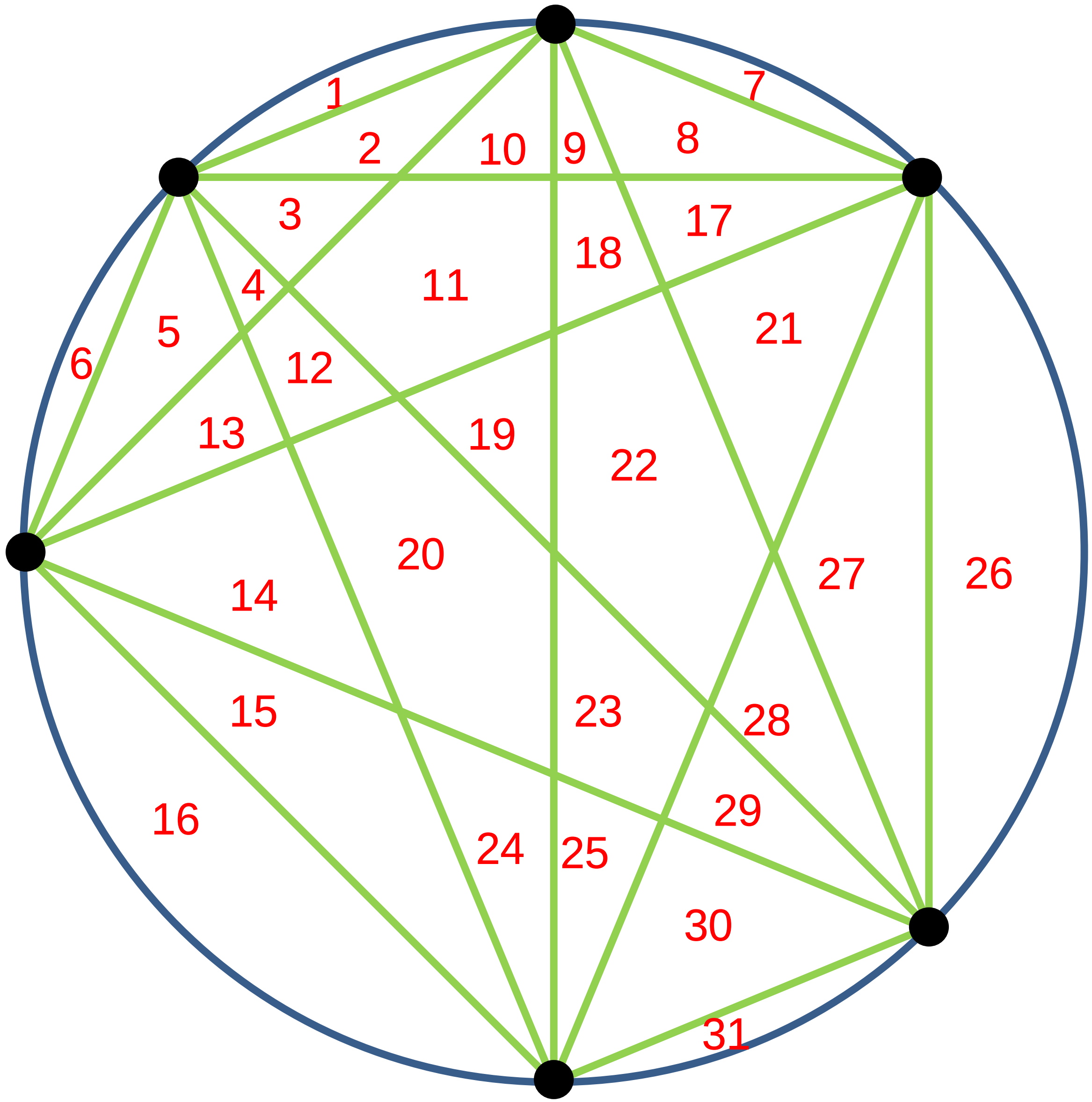
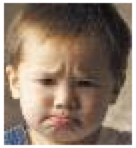


Solutions	
Points	Regions
2	2
3	4
4	8
5	16
6	**31**



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Extension: *Can you come up with a general formula for the number of regions created when "n" points placed on a circle are joined by chords?*

Always try to challenge the student who finishes early

General Equation for Chords/ regions in a circle

$$\frac{n(n-1)(n-2)(n-3)}{24}+\frac{n(n-1)}{2}+1$$

Investigation in Number

It looks like every odd number greater than 1 be expressed as the sum of a power of 2 and a prime number

e.g.

$3 = 2^0 + 2$

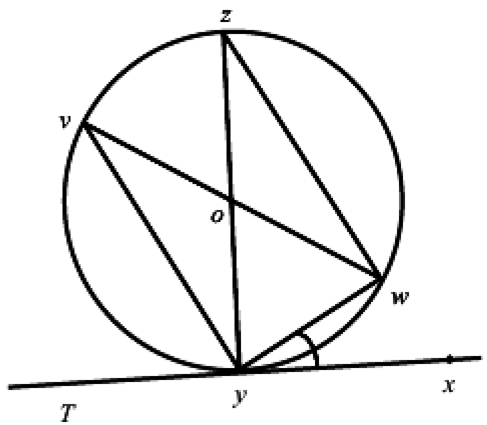
$5 = 2^1 + 3$

$7 = 2^2 + 3$ *or* $2^1 + 5$

Can we say it holds true forever? Why?

$$
\begin{aligned}
3 &= 2^0 + 2 \\
5 &= 2^1 + 3 \\
7 &= 2^2 + 3 \\
9 &= 2^2 + 5 \\
11 &= 2^3 + 3 \\
13 &= 2^3 + 5 \\
15 &= 2^3 + 7 \\
17 &= 2^2 + 13 \\
19 &= 2^4 + 3 \\
&\vdots \\
51 &= 2^5 + 19 \\
&\vdots \\
125 &= 2^6 + 61 \\
127 &= ? \\
129 &= 2^5 + 97 \\
131 &= 2^7 + 3
\end{aligned}
$$

Proving in Geometry



T is a tangent to the circle and o is the centre of the circle.

$|\angle xyw| = 40°$.

(i) Find $|\angle wvy|$.

(ii) Using congruent triangles or otherwise, prove $|zw| = |vy|$.

- ***Not Procedural (No "steps"!!!)***
- ***Problem Solving activity- "Write down what you see" is only a start***
- ***Teacher demonstrates in a linear fashion (zig zag in reality)***
- ***May be the first time students have ever encountered proof***
- ***Van Hiele level 4- difficult!***
- ***Takes time. 1 Q or 8 Q????***

Number: The sum of any two odd numbers is an even number.
Can you prove that this is true??

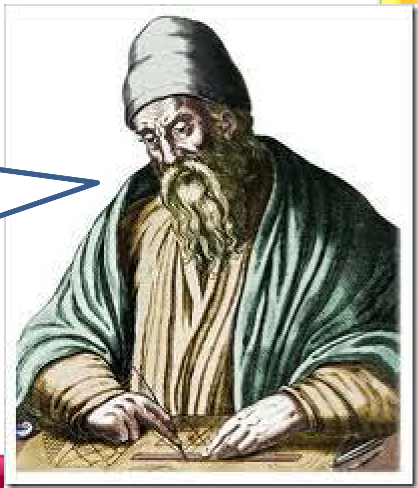
Equations: $If \quad 3(x-2)=42 \quad then \quad x=16.$

Inequalities: If Tim buys two shirts for just over €60,
can you prove that at least one of the shirts
cost more than €30??

Coordinate Geometry: Prove that triangles ABC and RST are congruent where the vertices are A(2,6), B(5,5), C(3,3) and R(9,5), S(8,2), T(6,4).

Transposing: If y=mx +c prove that x = (y-c)/m

Don't wait until
Geometry
to teach Proof!!!
It's a habit of mind!!!

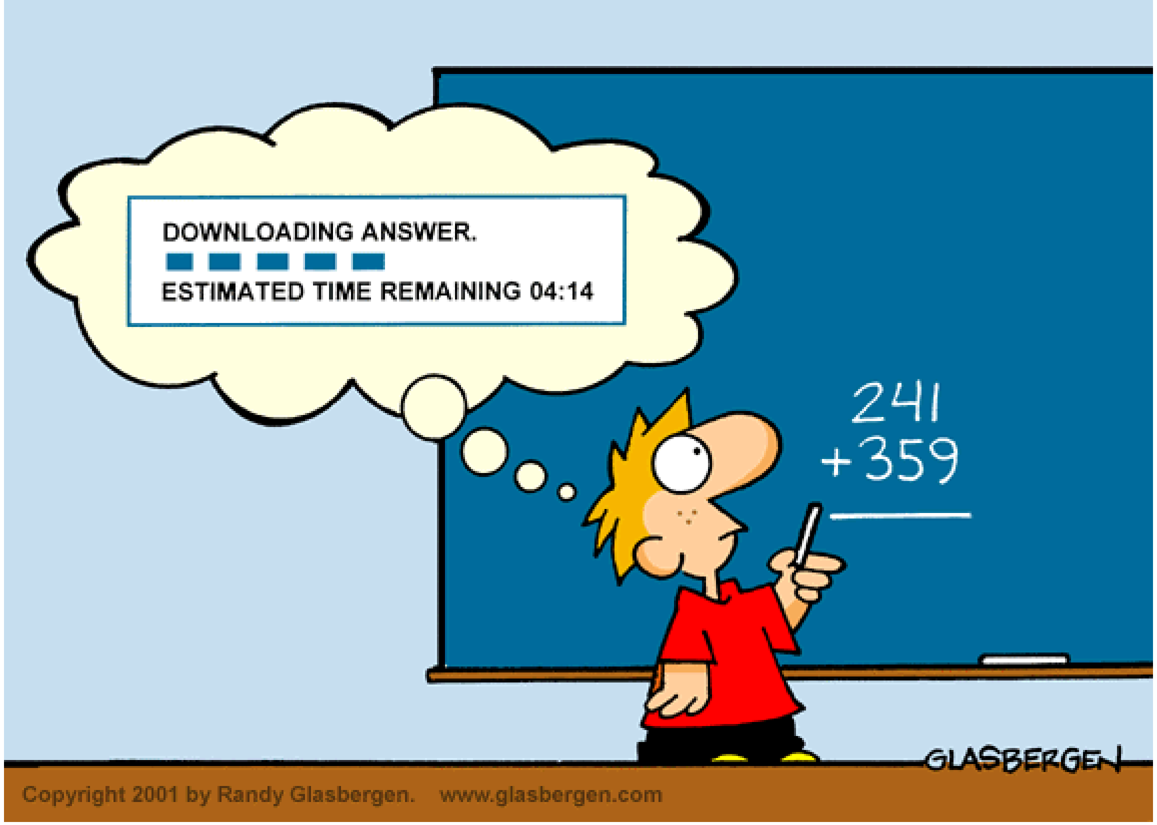


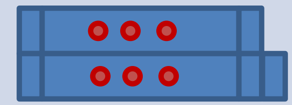
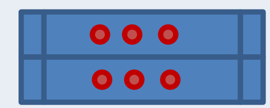
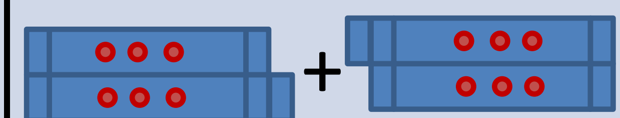
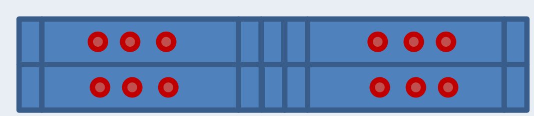
Number:

The sum of any two odd numbers is an even number.

Can you prove that this is true??

Can you do it in ***two*** different ways?



Proof using Words	Proof using algebra	Proof using pictures
Odd numbers are the numbers that if you group them in twos, there's one left over.	Odd numbers are the numbers of the form $2n + 1$, where n is a whole number.	Odd numbers are of the form 
Even numbers are the numbers that if you group them in twos, there's none left over.	Even numbers are the numbers of the form $2n$, where n is a whole number.	Even numbers are of the form 
If you add two odd numbers, the two ones are left over will make another group of two.	If you add two odd numbers, you get $(2k + 1) + (2m+1) = (2k +2m) + (1+1)$ $= 2(k + m + 1)$	If you add two odd numbers, 
The resulting number can be grouped by twos with none left over and, thus, is an even number	The resulting number is of the form $2n$ and, thus, is an even number	The result is 

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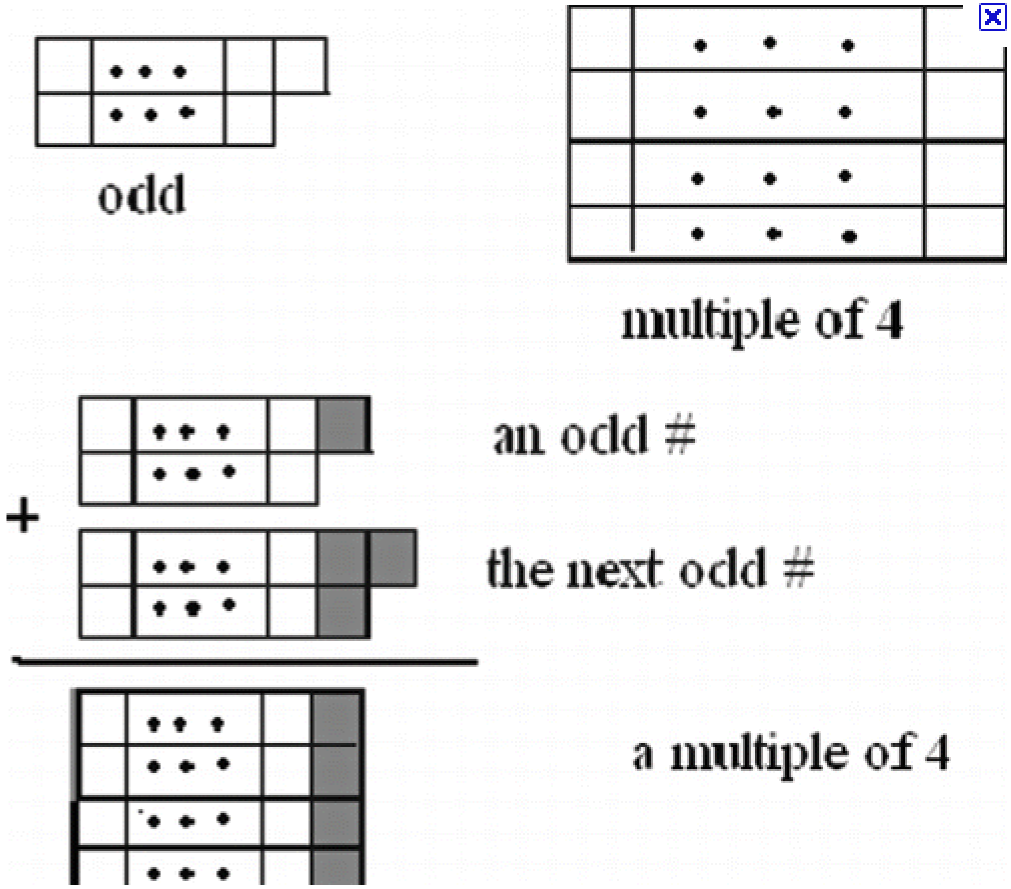
<u>Extension1</u> *The product of any two even numbers is an even number. Can you prove that this is true?? E.g. 4 x 6 =24,*

16 x 12=192 etc

<u>Extension2</u> *The sum of any two consecutive odd numbers is always a multiple of 4. Can you prove that this is true?? E.g. 5+7=12,*

19+21=40 etc


odd
multiple of 4
an odd #
+
the next odd #
a multiple of 4

Show that if you multiply three consecutive positive integers then the number you get is evenly divisible by 6.

Pair 1 "We multiplied 1 X 2 X 3 =6, 2 X 3 X 4 = 24, 10 X 11 X 12 = 1320 which can all be divided by 6. Then we tried 74 X 75 X 76 = 421800 which also works. It must work for all numbers so"

Let students compare and contrast different methods of proof

Work with proofs that need to be rectified

Show that if you multiply three consecutive positive integers then the number you get is evenly divisible by 6.

Pair 1 ***"We multiplied 1 X 2 X 3 =6, 2 X 3 X 4 = 24, 10 X 11 X 12 = 1320 which can all be divided by 6. Then we tried 74 X 75 X 76 = 421800 which also works. It must work for all numbers so"***

Pair 2 ***"We looked at consecutive numbers from 1 to 10 i.e. 1 2 3 4 5 6 7 8 9 10. When you take them in threes we noticed there is always one of them divisible by 3 and one or two of them divisible by 2. We think this has something to do with it"***

Work with proofs that need to be rectified

Purpose: Let's Define Some Terms!!

CONVERSE

Proof by Contradiction

Corollary

If and only if

Axiom

Implies

Introducing Indirect Proof: Munster game?

Paul and Mike are driving past Thomond Park. The floodlights are on.

Paul: *Are Munster playing tonight?*

Mike: *I don't think so. If a game were being played right now we would see or hear a big crowd but the stands are empty and there isn't any noise.*

Reduction ad Absurdum: Proof by Contradiction

Introducing Indirect Proof

Sarah left her house at 9:30 AM and arrived at her aunts house 80 miles away at 10:30 AM.

Use an indirect proof to show that Sarah exceeded the 55 mph speed limit.

Proof by Contradiction: Inequalities

If Tim buys two shirts for just over €60, can you prove that at least one of the shirts cost more than €30??

$$i.e. \quad If \;\; x + y > 60 \;\; \text{then either } x > 30 \text{ or } y > 30$$

Assume neither shirt costs more than €30

$$x \leq 30$$

$$+ \quad y \leq 30 \quad +$$

$$\Rightarrow x + y \leq 60$$

$\Rightarrow$ This is a contradiction since we know Tim spent more than €60

$\Rightarrow$ Our original assumption must be false

$\Rightarrow$ At least one of the shirts had to have cost more than €30

QED

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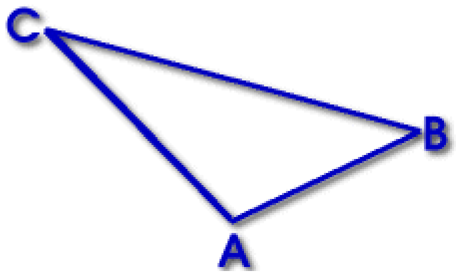
Geometry : Proof by Contradiction

Triangle ABC has no more than one right angle.
Can you complete a proof by contradiction for this statement?



Assume $\angle A$ and $\angle B$ are right angles

We know $\angle A + \angle B + \angle C = 180^0$

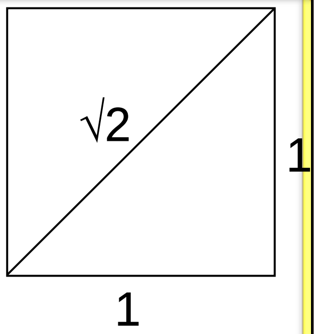
By substitution $90^0 + 90^0 + \angle C = 180^0$

$\therefore$ $\angle C = 0^0$ which is a contradiction

$\therefore$ $\angle A$ and $\angle B$ cannot both be right angles

$\Rightarrow$ A triangle can have at most one right angle

Proof by Contradiction: The Square Root of 2 is Irrational



To prove that √2 is irrational

Assume the contrary: √2 is rational

i.e. there exists integers p and q with no common factors such that:

$$\frac{p}{q} = \sqrt{2}$$ *(Square both sides)*

$$\Rightarrow \frac{p^2}{q^2} = 2$$ *(Multiply both sides by q^2)*

$$\Rightarrow p^2 = 2q^2$$

(......it's a multiple of 2)

(......even² = even)

$$\frac{p}{q} = \sqrt{2}$$

$$\Rightarrow \frac{p^2}{q^2} = 2$$

$$\Rightarrow p^2 = 2q^2$$

$$\Rightarrow p^2 \text{ is even}$$

$$\Rightarrow p \text{ is even}$$

$$\Rightarrow p \text{ is even}$$

$$\therefore p = 2k \text{ for some } k$$

$$\text{If } p = 2k$$

$$\Rightarrow p^2 = 2q^2 \text{ becomes } (2k)^2 = 2q^2$$

$$\Rightarrow 4k^2 = 2q^2$$ *(Divide both sides by 2)*

$$\Rightarrow 2k^2 = q^2$$

Then similarly $q = 2m$ for some m

$$\Rightarrow \frac{p}{q} = \frac{2k}{2m} \Rightarrow \frac{p}{q} \text{ has a factor of 2 in common.}$$

This contradicts the original assumption.

<u>√2 is irrational</u> **Q.E.D.**

13:44

To prove that √2 is irrational

√2

Assume the contrary: √2 is rational

That is, there exists integers p and q with no common factors(except 1) such that:

$$\frac{p}{q} = \sqrt{2}$$

$$\Rightarrow \frac{p^2}{q^2} = 2 \quad \Rightarrow p^2 = 2q^2 \Rightarrow p \text{ is even}$$

$p = 2k$ for some k.

$p = 2k \Rightarrow$ … comes $(2k)^2 = 2q^2$

$$\Rightarrow 4k^2 = 2q^2$$

$\Rightarrow 2k^2 = q^2$ So $q = 2m$ for some m.

$\Rightarrow \frac{p}{q} = \frac{2k}{2m} \Rightarrow \frac{p}{q}$ *have a factor of 2 in common.*

This contradicts the original assumption.

√2 is irrational. QED

13:17